ثمرة

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Calculus 1 – Derivative

(Section 3.1)

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Basic Derivatives Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

1

$$\frac{d}{dx}(x) = 1$$

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

3.1 Derivatives of Polynomials and Exponential Functions

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0 = 0$$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

$\frac{d}{dx}(e) =$, $\frac{d}{dt}(\pi) =$

$\frac{d}{dz}(2^3) =$, $\frac{d}{dt}(-5) =$

Power Functions

We next look at the functions $f(x) = x^n$, where n is a positive integer. If $n = 1$, the graph of $f(x) = x$ is the line $y = x$, which has slope 1. (See Figure 2.) So

1 $$\frac{d}{dx}(x) = 1$$

2 $$\frac{d}{dx}(x^2) = 2x \qquad \frac{d}{dx}(x^3) = 3x^2$$

$\frac{d}{dx}(x^{-2}) =$

$\frac{d}{dx}(x^{-\frac{1}{2}}) =$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

$\frac{d}{dx}(3) =$, $\frac{d}{dx}(-1/3) =$, $\frac{d}{dx}(0) =$, $\frac{d}{dx}(e) =$

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$\frac{d}{dx}(x) =$, $\frac{d}{dx}(x^{-\frac{2}{3}}) =$

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

$\frac{d}{dx}(4x^3) =$, $\frac{d}{dx}(-1/2\, x^{-2}) =$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

$\frac{d}{dx}(4e^x) =$

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

EXAMPLE 1

(a) If $f(x) = x^6$, then $f'(x) = 6x^5$.

(b) If $y = x^{1000}$, then $y' = 1000x^{999}$.

(c) If $y = t^4$, then $\frac{dy}{dt} = 4t^3$.

(d) $\frac{d}{dr}(r^3) = 3r^2$ ■

e) $$\frac{d}{dx}\left(\frac{1}{x}\right) =$$

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f) $$\frac{d}{dx}\sqrt{x} =$$

رح نشوفها وأيد تأكد إنك عارف شلون اشتقيناها

EXAMPLE 2 Differentiate:

(a) $f(x) = \frac{1}{x^2}$

(b) $y = \sqrt[3]{x^2}$

EXAMPLE 3 Find equations of the tangent line and normal line to the curve $y = x\sqrt{x}$ at the point $(1, 1)$. Illustrate by graphing the curve and these lines.

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

EXAMPLE 4

(a) $\frac{d}{dx}(3x^4) =$

(b) $\frac{d}{dx}(-x) =$

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$(f + g)' = f' + g'$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

EXAMPLE 5

$$\frac{d}{dx}(x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5)$$

EXAMPLE 6 Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

Exponential Functions

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

EXAMPLE 8 If $f(x) = e^x - x$, find f' and f''. Compare the graphs of f and f'.

3–32 Differentiate the function.

3. $f(x) = 2^{40}$

5. $f(x) = 5.2x + 2.3$

7. $f(t) = 2t^3 - 3t^2 - 4t$

9. $g(x) = x^2(1 - 2x)$

11. $g(t) = 2t^{-3/4}$

13. $F(r) = \dfrac{5}{r^3}$

3–32 Differentiate the function.

15. $R(a) = (3a + 1)^2$

17. $S(p) = \sqrt{p} - p$

3–32 Differentiate the function.

19. $y = 3e^{x} + \dfrac{4}{\sqrt[3]{x}}$

25. $j(x) = x^{2.4} + e^{2.4}$

33–36 Find an equation of the tangent line to the curve at the given point.

33. $y = 2x^3 - x^2 + 2, \quad (1, 3)$

33–36 Find an equation of the tangent line to the curve at the given point.

34. $y = 2e^x + x, \quad (0, 2)$

33–36 Find an equation of the tangent line to the curve at the given point.

35. $y = x + \dfrac{2}{x}, \quad (2, 3)$

33–36 Find an equation of the tangent line to the curve at the given point.

36. $y = \sqrt[4]{x} - x, \quad (1, 0)$

56. For what value of x does the graph of $f(x) = e^x - 2x$ have a horizontal tangent?

7. [5 + 5 = 10 pts.] Let $f(x) = e^x - 2x$.

(a) Find the point on the curve of f where the tangent line is horizontal.

(b) Using part (a) find an equation of this horizontal tangent line.

57. Show that the curve $y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2.

58. Find an equation of the tangent line to the curve $y = x^4 + 1$ that is parallel to the line $32x - y = 15$.

4 Theorem If f is differentiable at a, then f is continuous at a.

NOTE The converse of Theorem 4 is false; that is, there are functions that are continuous but not differentiable. For instance, the function $f(x) = |x|$ is continuous at 0 because

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} |x| = 0 = f(0)$$

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for f to be differentiable at $x = a$, then

1) $\lim_{x \to a} = f(a)$ "continuous"

2) $f'_{+}(a) = f'_{-}(a)$

71. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ x + 1 & \text{if } x \geqslant 1 \end{cases}$$

Is f differentiable at 1? S~~ketch the graphs of f and f~~$'$.

72. At what numbers is the following function g differentiable?

$$g(x) = \begin{cases} 2x & \text{if } x \leqslant 0 \\ 2x - x^2 & \text{if } 0 < x < 2 \\ 2 - x & \text{if } x \geqslant 2 \end{cases}$$

81. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leqslant 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Find the values of m and b that make f differentiable everywhere.

Using Definition of Derivative to Evaluate a Limit

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

1) $\displaystyle \lim_{x \to 0} \frac{e^{x} - 1}{x}$

83. Evaluate $\lim_{x \to 1} \dfrac{x^{1000} - 1}{x - 1}$.

Question 4. Evaluate the following limits:

(a) $\lim_{x \to 1} \frac{x^{2017} - 1}{x - 1}$

Question 4. Evaluate the following limits:

(d) $\lim_{x \to 16} \dfrac{\sqrt[4]{x} - 2}{x - 16}$

(e) $\lim_{x \to 0} \frac{e^x - 1}{x - 1}$

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Calculus 1 – Product and Quotient Rule

(Section 3.2)

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The Product Rule

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

EXAMPLE 1

(a) If $f(x) = xe^x$, find $f'(x)$.

(b) Find the nth derivative, $f^{(n)}(x)$.

EXAMPLE 2 Differentiate the function $f(t) = \sqrt{t}\,(a + bt)$.

EXAMPLE 3 If $f(x) = \sqrt{x}\, g(x)$, where $g(4) = 2$ and $g'(4) = 3$, find $f'(4)$.

The Quotient Rule

مهم الترتيب شوف وين مكان اشتقاق المقام ووين مكان اشتقاق البسط وواحد قبل الطرح
وواحد بعد الطرح (لا تعكس أماكنهم لأنه الطرح غير إبدالي)

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,\frac{d}{dx}[f(x)] - f(x)\,\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

EXAMPLE 4 Let $y = \dfrac{x^2 + x - 2}{x^3 + 6}$. Then $y' = ?$

$$y' = \frac{(x^3 + 6)\,\frac{d}{dx}(x^2 + x - 2) - (x^2 + x - 2)\,\frac{d}{dx}(x^3 + 6)}{(x^3 + 6)^2}$$

$$= \frac{(x^3 + 6)(2x + 1) - (x^2 + x - 2)(3x^2)}{(x^3 + 6)^2}$$

$$= \frac{(2x^4 + x^3 + 12x + 6) - (3x^4 + 3x^3 - 6x^2)}{(x^3 + 6)^2}$$

$$= \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3 + 6)^2}$$

Table of Differentiation Formulas

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$
$(cf)' = cf'$	$(f + g)' = f' + g'$	$(f - g)' = f' - g'$
$(fg)' = fg' + gf'$	$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$	

EXAMPLE 5 Find an equation of the tangent line to the curve $y = e^x/(1 + x^2)$ at the point $\left(1, \frac{1}{2}e\right)$.

3–26 Differentiate.

3. $f(x) = (3x^2 - 5x)e^x$

4. $g(x) = \left(x + 2\sqrt{x}\right)e^x$

6. $y = \dfrac{e^x}{1 - e^x}$

3–26 Differentiate.

5. $y = \dfrac{x}{e^x}$

7. $g(x) = \dfrac{1 + 2x}{3 - 4x}$

8. $G(x) = \dfrac{x^2 - 2}{2x + 1}$

3–26 Differentiate.

9. $H(u) = \left(u - \sqrt{u}\right)\left(u + \sqrt{u}\right)$

10. $J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$

12. $f(z) = (1 - e^z)(z + e^z)$

3–26 Differentiate.

14. $y = \dfrac{\sqrt{x}}{2 + x}$

3–26 Differentiate.

15. $y = \dfrac{t^3 + 3t}{t^2 - 4t + 3}$

17. $y = e^p\left(p + p\sqrt{p}\right)$

23. $f(x) = \dfrac{x^2 e^x}{x^2 + e^x}$

27–30 Find $f'(x)$ and $f''(x)$.

27. $f(x) = (x^3 + 1)e^x$

27–30 Find $f'(x)$ and $f''(x)$.

28. $f(x) = \sqrt{x}\,e^x$

3–26 Differentiate.

11. $F(y) = \left(\dfrac{1}{y^2} - \dfrac{3}{y^4}\right)(y + 5y^3)$

33–34 Find equations of the tangent line and normal line to the given curve at the specified point.

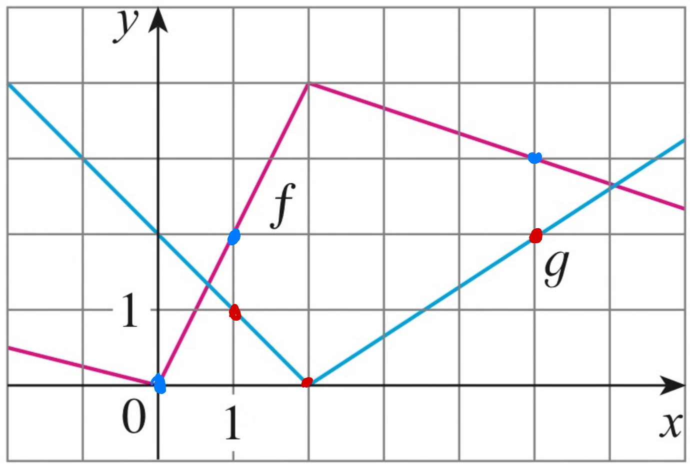
33. $y = 2xe^x, \quad (0, 0)$

47. If $g(x) = xf(x)$, where $f(3) = 4$ and $f'(3) = -2$, find an equation of the tangent line to the graph of g at the point where $x = 3$.

49. If f and g are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$.

(a) Find $u'(1)$.

(b) Find $v'(5)$.



2. [10 pts.] Find **equations** of the tangent lines to the curve $f(x) = \dfrac{x-1}{x+1}$ that are **parallel** to the line $x - 2y = 2$.

We have $f^{'}(x) = \dfrac{2}{(x+1)^2}$. Now, any tangent line to the curve has slope $= \dfrac{2}{(x+1)^2} = \dfrac{1}{2}$. This implies that $x = 1$ or $x = -3$. Therefore, the equation of the first tangent line is $y - 0 = \dfrac{1}{2}(x-1)$ and the equation of the second tangent line is $y - 2 = \dfrac{1}{2}(x+3)$.

8. [10 pts.] Find an equation of the tangent line to the graph of the function $f(x) = \dfrac{e^x}{x+1} + x + 3$ at $x = 0$.

We have $f'(x) = \dfrac{xe^x}{(x+1)^2} + 1$. It is clear that $f(0) = 4$ and $f'(0) = 1$. Therefore, an equation of the tangent line to the graph of the function f at $x = 0$ is given by $y = x + 4$.

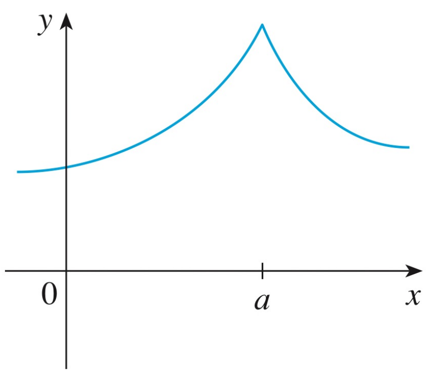
7. [10 pts.] Find the points on the graph: $y = \dfrac{x}{x^2+1}$ where the tangent line is horizontal.

We have $\boxed{\dfrac{dy}{dx} = \dfrac{1(x^2+1) - x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}}$.

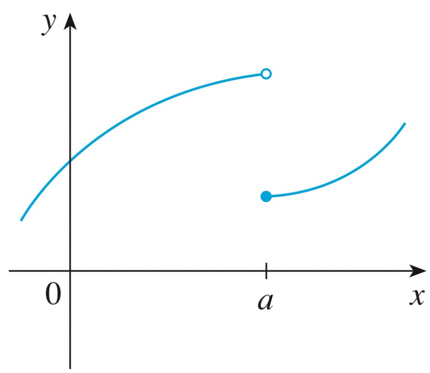
Thus, $\dfrac{dy}{dx} = 0$ when $\boxed{x = \pm 1}$. So the given curve has horizontal tangents when $\boxed{x = \pm 1}$. The corresponding points are $\boxed{(\pm 1, \pm 1/2)}$.

How Can a Function Fail To Be Differentiable?

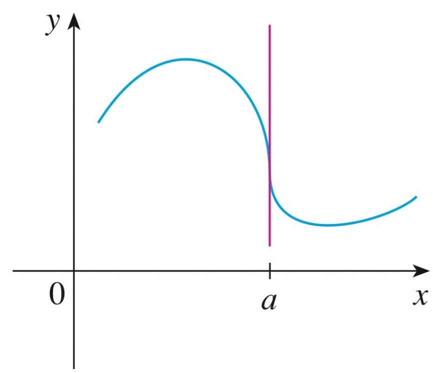
not differentiable يعني متى تكون



(a) A corner



(b) A discontinuity



(c) A vertical tangent

64. The **left-hand** and **right-hand derivatives** of f at a are defined by

$$f'_-(a) = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$$

and $$f'_+(a) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

if these limits exist. Then $f'(a)$ exists if and only if these one-sided derivatives exist and are equal.

(a) Find $f'_-(4)$ and $f'_+(4)$ for the function

$$f(x) = \begin{cases} 0 & \text{if } x \leqslant 0 \\ 5 - x & \text{if } 0 < x < 4 \\ \dfrac{1}{5 - x} & \text{if } x \geqslant 4 \end{cases}$$

(b) Sketch the graph of f. "معطى الصفحة القادمة"

(c) Where is f discontinuous?

(d) Where is f not differentiable?

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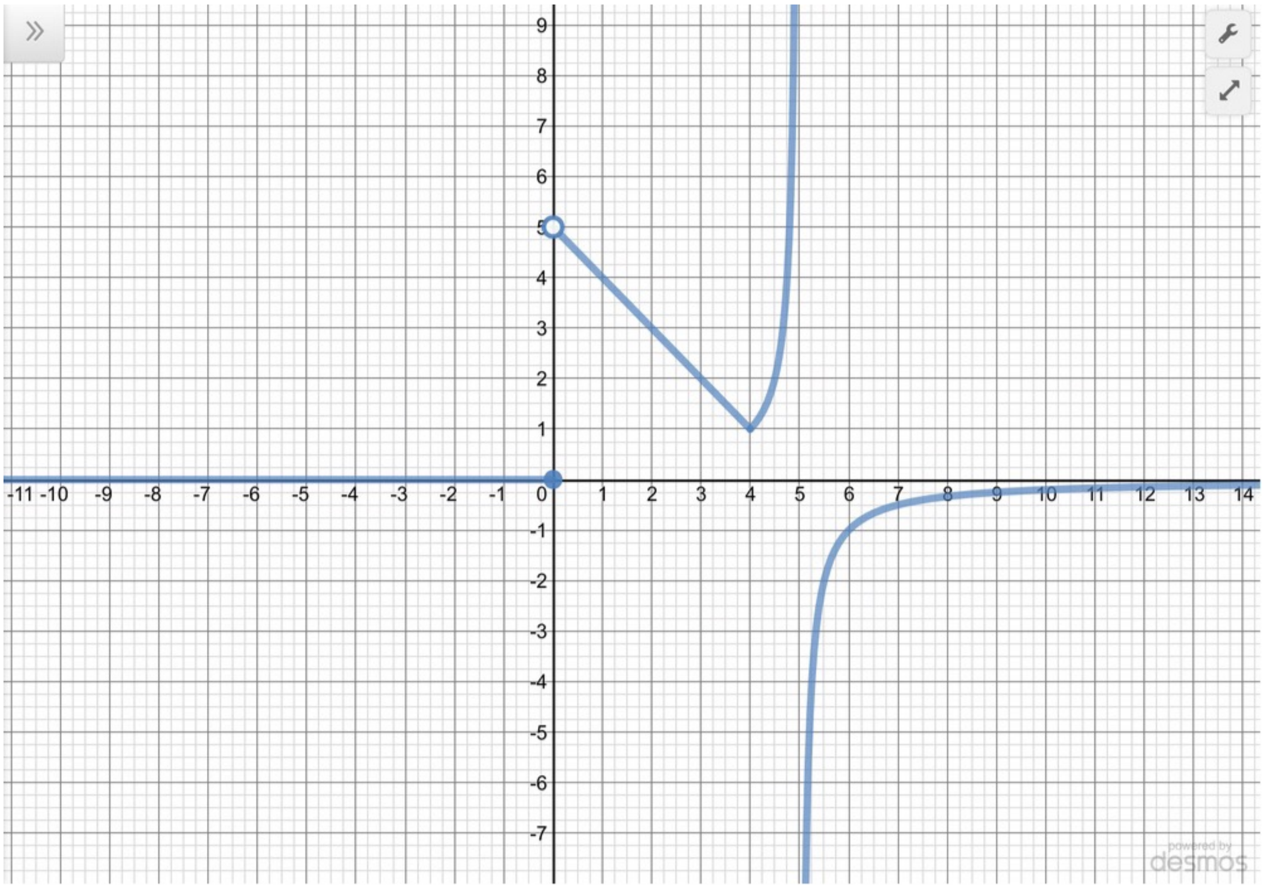

(c) Where is f discontinuous?

(d) Where is f not differentiable?

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Calculus 1 – Derivatives of Trigonometric

(Section 3.3)

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Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

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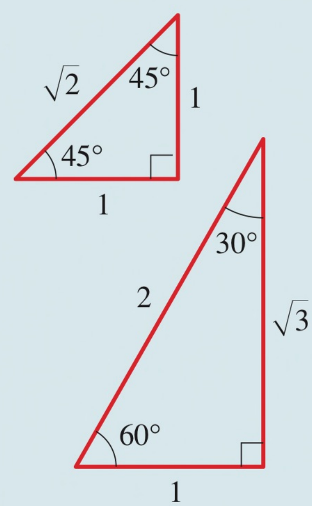
SPECIAL VALUES OF THE TRIGONOMETRIC FUNCTIONS

The following values of the trigonometric functions are obtained from the special triangles.

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0	0	1	0	—	1	—
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	—	1	—	0



3.3 Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

EXAMPLE 1 Differentiate $y = x^2 \sin x$.

Table of Differentiation Formulas

$$\frac{d}{dx}(c) = 0 \qquad \frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dx}(e^x) = e^x$$

$$(cf)' = cf' \qquad (f+g)' = f' + g' \qquad (f-g)' = f' - g'$$

$$(fg)' = fg' + gf' \qquad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0	0	1	0	—	1	—
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	—	1	—	0

FUNDAMENTAL TRIGONOMETRIC IDENTITIES

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Even-Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

Remark :-

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PROPERTIES OF LIMITS

These properties require that the limit of $f(x)$ and $g(x)$ exist

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)\, g(x)] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \text{ if } \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$$

EXAMPLE 4 Find the 27th derivative of $\cos x$.

EXAMPLE 1

(a) If $f(x) = xe^x$, find $f'(x)$.

(b) Find the nth derivative, $f^{(n)}(x)$.

Answer in section 3.2

1–16 Differentiate.

1. $f(x) = x^2 \sin x$

2. $f(x) = x \cos x + 2 \tan x$

5. $y = \sec\theta \tan\theta$

1–16 Differentiate.

6. $g(\theta) = e^{\theta}(\tan\theta - \theta)$

1–16 Differentiate.

8. $f(t) = \dfrac{\cot t}{e^t}$

1–16 Differentiate.

9. $y = \dfrac{x}{2 - \tan x}$

11. $f(\theta) = \dfrac{\sin \theta}{1 + \cos \theta}$

31. (a) Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan x - 1}{\sec x}$$

(b) Simplify the expression for $f(x)$ by writing it in terms of $\sin x$ and $\cos x$, and then find $f'(x)$.

(c) Show that your answers to parts (a) and (b) are equivalent.

31. (a) Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan x - 1}{\sec x}$$

(b) Simplify the expression for $f(x)$ by writing it in terms of $\sin x$ and $\cos x$, and then find $f'(x)$.

(c) Show that your answers to parts (a) and (b) are equivalent.

21–24 Find an equation of the tangent line to the curve at the given point.

22. $y = e^x \cos x, \quad (0, 1)$

33–34 For what values of x does the graph of f have a horizontal tangent?

34. $f(x) = e^x \cos x$

EXAMPLE 2 Differentiate $f(x) = \dfrac{\sec x}{1 + \tan x}$. For what values of x does the graph of f have a horizontal tangent?