Thamrah

Kuwait University

Calculus A – Basic

(المدخل الى كالكولس 1)

For Contact and Support:

YouTube: Precalculusq8 Twitter: Precalculusq8

* العناويـن *

1- Common Factors.

2- Difference of Squares.

3- Special Products

4- Absolute Value.

5- Inequalities.

6- Exponent and Roots Rules.

7- LCM and Complex Fractions.

8- Multiple by Conjugate.

9- General Algebra Notes.

10- Trigonometry Basics.

11- Factoring Trinomials

12- Solving Equations.

13- Domain

* 1. Common Factor *

Rule: Factor out the greatest Common or variable.

Examples:-

- $6x + 9 = 3(2x + 3)$
- $x^3 + x^2 = x^2(x + 1)$

---

* 2. Difference of squares *

a) $a^2 - b^2 = (a - b)(a + b)$

Examples:-

- $x^2 - 9 = (x - 3)(x + 3)$
- $x^2 - 1 = (x - 1)(x + 1)$

3. Special Products

b) $a^3+b^3=(a+b)(a^2-ab+b^2)$

c) $a^3-b^3=(a-b)(a^2+ab+b^2)$

Examples:-

- $x^3+8=(x+2)(x^2-2x+4)$

- $x^3-1=(x-1)(x^2+x+1)$

d) $(x+y)^3=x^3+3x^2y+3xy^2+y^3$

e) $(x-y)^3=x^3-3x^2y+3xy^2-y^3$

Examples:-

- $(2+x)^3=2^3+3(2)^2(x)+3(2)(x)^2+x^3$

  $8+12x+6x^2+x^3$

- $(x-1)^3=x^3-3x^2(1)+3(x)(1^2)-1^3$

  x^3-3x^2+3x-1

f) $(x+y)^2 = x^2 + 2xy + y^2$

g) $(x-y)^2 = x^2 - 2xy + y^2$

Examples:-

$(x+3)^2 = x^2 + 6x + 9$

$(x-3)^2 = x^2 - 6x + 9$

---

* 4. Absolute Value *

$$|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example:

$$|x-5| = \begin{cases} x-5 & \text{if } x \geqslant 5 \\ -(x-5) & \text{if } x < 5 \end{cases}$$

* 5. Inequalities *

Rule: When multiplying or dividing by a negative number, Flip the inequality Sign.

Examples:-

- $-2x > 6 \rightarrow x < -3$

- $x > 4$, $x < 2$, $x \geq 0$, $x > -2$
$x < -5$, $-3 \leq x < 10$

---

* 6. Exponent and Root Rules *

- $x^a \cdot x^b = x^{a+b}$
- $\frac{x^a}{x^b} = x^{a-b}$
- $(x^a)^b = x^{ab}$
- $x^{-a} = \frac{1}{x^a}$
- $x^{1/n} = \sqrt[n]{x}$

Examples:-

$x^3 \cdot x^2 = x^5$

$(x^2)^3 = x^6$

$x^{-2} = \frac{1}{x^2}$

* 7. LCM and Complex Fractions *

Rule: Use the Least Common multiple (LCM) of small denominators to simplify or combine terms.

Examples: -

$$\frac{1}{x} + \frac{2}{x+1} = \frac{(x+1)+2x}{x(x+1)} = \frac{3x+1}{x(x+1)}$$

"Fractions" فهم الكسور

PROPERTIES OF FRACTIONS

Property	Example	Description
1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$	When **multiplying fractions**, multiply numerators and denominators.
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$	When **dividing fractions**, invert the divisor and multiply.
3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$	$\frac{2}{5} + \frac{7}{5} = \frac{2+7}{5} = \frac{9}{5}$	When **adding fractions** with the **same denominator**, add the numerators.
4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{2}{5} + \frac{3}{7} = \frac{2 \cdot 7 + 3 \cdot 5}{35} = \frac{29}{35}$	When **adding fractions** with **different denominators**, find a common denominator. Then add the numerators.
5. $\frac{ac}{bc} = \frac{a}{b}$	$\frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3}$	**Cancel** numbers that are **common factors** in numerator and denominator.
6. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$	$\frac{2}{3} = \frac{6}{9}$, so $2 \cdot 9 = 3 \cdot 6$	**Cross-multiply**.

1) $\dfrac{\frac{4}{5}}{2} = \dfrac{4}{5} \div 2$

$= \dfrac{4}{5} * \dfrac{1}{2} = \dfrac{4}{10} = \dfrac{2}{5}$

2) $\dfrac{3}{\frac{5}{7}} = 3 \div \dfrac{5}{7} \Rightarrow 3 * \dfrac{7}{5} = \dfrac{21}{5}$

3) $\dfrac{\frac{2}{3}}{\frac{4}{5}} = \dfrac{2}{3} \div \dfrac{4}{5} \Rightarrow \dfrac{2}{3} * \dfrac{5}{4} =$

$\dfrac{5}{6}$

* 8. Multiply by Conjugate *

Use $(a-b)(a+b) = a^2 - b^2$ to eliminate square roots form the numerator or denominator.

Example :-

$$\frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \frac{x-9}{(x-9)(\sqrt{x}+3)} = \frac{1}{\sqrt{x}+3}$$

---

* 9. General Algebra Notes *

- Don't cancel terms unless they are factors
- only distribute over addition / subtraction when appropriate.
- Be careful with signs:-
  - $-(x+3) = -x-3$
  - $(a+b)^2 = a^2 + 2ab + b^2$

wrong ×

$(a+b)^2 = a^2 + b^2$

* 10. Trigonometry Basics *

Function	Domain	Range
$\sin x$	$(-\infty, \infty)$	$[-1, 1]$
$\cos x$	$(-\infty, \infty)$	$[-1, 1]$
$\tan x$	$x \in R$, $x \neq \frac{\pi}{2} + n\pi$	$(-\infty, \infty)$

* Note : $R = (-\infty, \infty)$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

$$\csc = \frac{1}{\sin x}$$

$$\sec = \frac{1}{\cos x}$$

$$\cot = \frac{1}{\tan}$$

Special Values:-

x (degrees/radians)	sin x	cos x	tan x
$0^\circ / 0$	0	1	0
$30^\circ / \pi/6$	$1/2$	$\sqrt{3}/2$	$\frac{\sqrt{3}}{3}$
$45^\circ / \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60^\circ / \frac{\pi}{3}$	$\sqrt{3}/2$	$\frac{1}{2}$	$\sqrt{3}$
$90^\circ / \frac{\pi}{2}$	1	0	undefined

Factoring Trinomials

1) $x^2 + bx + c$

* ملاحظة مهمة *

إذا كان معامل الـ x بـ 1

① عددين ضربهم يساوي قيمة الثابت .

② مجموعهم يساوي العدد الذي في الوسط .

a) $x^2 + x - 72 = (x+9)(x-8)$

b) $x^2 - 6x - 16 = (x-8)(x+2)$

c) $x^2 - 3x + 2 = (x-2)(x-1)$

d) $x^2 + 3x + 2 = (x+2)(x+1)$

e) $x^2 + 7x - 8 = (x+8)(x-1)$

2) $ax^2 + bx + c$

* ملاحظة مهمة *

إذا كان معامل الـ x غير 1 " أسلوب التجربة والتحقق "

① عددين ضربهم يساوي قيمة الثابت .

② مجموع ضرب الأرقام الخارجية مع ضرب الأرقام الداخلية يساوي العدد الذي في الوسط .

$3x^2 + \underline{8x} + 4 = (3x + 2)(x + 2)$

$6x + 2x = 8x$ ✓ تحقق

$2x^2 - \underline{7x} - 4 = (2x + 1)(x - 4)$

$-8x + x = -7x$ ✓ تحقق

$2x^2 \underline{- 3x} - 2 = (2x + 1)(x - 2)$

$-4x + x = -3x$ ✓ تحقق

Factor the following

$$(x^2-9) = (x-3)(x+3)$$

$$(x^2-16) = (x-4)(x+4)$$

$$(y^2-49) = (y-7)(y+7)$$

$$(x^2+9) = x^2+9$$

ما تتحلل لأنه بينهم جمع

$$(x^2-7) = (x-\sqrt{7})(x+\sqrt{7})$$

$$(x^2-a^2) = (x-a)(x+a)$$

$$(x^2-1) = (x-1)(x+1)$$

$$(4t^2-9) = (2t-3)(2t+3)$$

$$(x^4-25) = (x^2-5)(x^2+5)$$

$$(x-4) = (\sqrt{x}-2)(\sqrt{x}+2)$$

غالبا نخليها مثل ما أهيا إلا إذا طلب مني factor أو بعد الfactor أقدر اختصرها مع شي ثاني

Simplify

3. $\frac{x-4}{x^2-4} \div \frac{2x^2-7x-4}{2x^2-3x-2} =$

$$\frac{x-4}{(x-2)(x+2)} \cdot \frac{2x^2-3x-2}{2x^2-7x-4}$$

$$\frac{x-4}{(x-2)(x+2)} \cdot \frac{(2x+1)(x-2)}{(2x+1)(x-4)} = \frac{1}{x+2}$$

خواص log

$$\boxed{\log_a x = b \iff a^b = x}$$

١ـ خاصية الضرب :ـ

$$\log_a(MN) = \log_a(M) + \log_a(N)$$

مثال :

$$\log_2(32) = \log_2(8.4) = \log_2(8) + \log_2 4$$

$$= 3 + 2 = 5$$

٢ـ خاصية القسمة :ـ

$$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$$

مثال :ـ

$$\log_5\left(\frac{25}{5}\right) = \log_5 25 - \log_5 5$$

$$= 2 - 1 = 1$$

٣- خاصية القوة :-

$$\log_a(M^k) = K.\log(M)$$

مثال :

$$\log_3 9^2 = 2\log_3 9 \Rightarrow 2.2 = 4$$

٤ـ علاقة الأسس و اللوغاريثم :-

$$a^{\log_a M} = M \quad , \quad \log_a (a)^K = K$$

مثال :-

$$10^{\log 7} = 7 \quad , \quad \log_2 (2^5) = 5$$

فكرة دمج وتوسيع اللوغاريثم

١ـ توسيع (Expand)

$$\log\left(\frac{3x}{y}\right) = \log(3x) - \log(y)$$

$$= \log 3 + \log x - \log(y)$$

٢ـ دمج (Single log)

$$2\log(a) + \log b - \log c$$

$$\log(a^2) + \log b - \log c$$

$$\log(a^2 b) - \log c$$

$$\log\left(\frac{a^2 b}{c}\right)$$

" جدول ملخص الـ log "

القانون	الخاصية
$\log_a(MN) = \log_a(M) + \log_a(N)$	الضرب
$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$	القسمة
$\log_a(M^k) = K \cdot \log_a(M)$	القوة
$a^{\log_a M} = M$, $\log_a(a^K) = K$	الأساس واللوغاريتم
$\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$	تغير الأساس

حل المعادلات :-

1) $3x - 5 = 7$

$\Rightarrow 3x = 12 \Rightarrow x = 4$

2) $e^{2x} - 7 = 5$

$= e^{2x} = 5 + 7 \Rightarrow e^{2x} = 12$

$\ln e^{2x} = \ln 12 \Rightarrow 2x = \ln 12$

$\Rightarrow x = \frac{\ln 12}{2}$

3) $\ln(x+3) = 2$

$\Rightarrow e^{\ln(x+3)} = e^{2} \Rightarrow x + 3 = e^{2}$

$\Rightarrow x = e^{2} - 3$

4) $\dfrac{5}{x} = 10$

$\Rightarrow 5 = 10x \Rightarrow x = \dfrac{5}{10} = \dfrac{1}{2}$

5) $\dfrac{3}{x+3} = 6$

$\Rightarrow 3 = 6(x+3)$

$\Rightarrow 3 = 6x + 18 \Rightarrow -18 + 3 = 6x$

$\Rightarrow 6x = -15 \Rightarrow x \Rightarrow \dfrac{-15}{6}$

6) $\dfrac{1}{x} + \dfrac{1}{2} = 1$

$\dfrac{2+x}{2x} = 1 \Rightarrow 2 + x = 2x$

$\Rightarrow 2 = 2x - x \Rightarrow x = 2$

7) $|x-4| = 3$

$x-4=3 \Rightarrow x=7$

or

$x-4=-3 \Rightarrow x=1$

8) $\sqrt{x+1} = 5$

$\Rightarrow x+1 = 25 \Rightarrow x = 24$

9) $x^2 - 9 = 0$

$\Rightarrow x^2 = 9 \Rightarrow \sqrt{x^2} = \sqrt{9}$

$|x| = 3 \Rightarrow x = 3$ or $x = -3$

10) $x^2 + 5x + 4 = 0$

$(x+4)(x+1) = x = -4$

$x = -1$

How to Solve

١ـ المعادلات الجبرية :ـ

$2x+5=11 \Rightarrow 2x=6$

$x=3$

٢ـ المعادلات الكسرية :ـ

$3=\frac{x+1}{2} \Rightarrow 6=x+1$

$\Rightarrow x=5$

٣ـ المعادلات الجذرية :ـ

$5=\sqrt{x+4} \Rightarrow 25=x+4$

$x=21$

٤ ـ المعادلات الأسية :ـ

• $2^x = 16 \Rightarrow \log_2 2^x = \log_2 16$

$\Rightarrow x = 4$

• $e^x - 3 = 5 \Rightarrow e^x = 8$

$\ln e^x = \ln 8 \Rightarrow x = \ln 8$

٥ ـ المعادلات اللوغارتمية :ـ

$\log(x-1) = 2 \Rightarrow 10^{\log(x-1)} = 10^2$

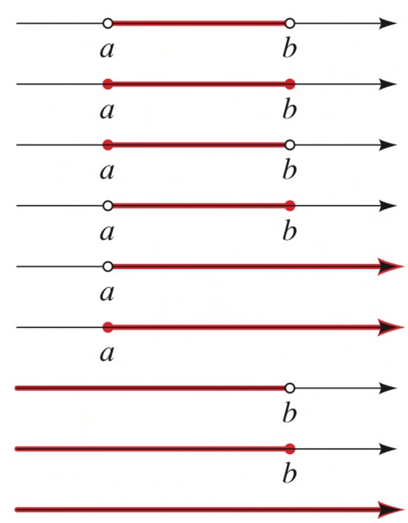
$\Rightarrow x - 1 = 100$

$\Rightarrow x = 101$

Intervals

* مهم *

Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \le x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	



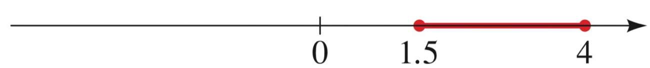
رقم الصغير دائما ينكتب على اليسار

EXAMPLE 5 ■ Graphing Intervals

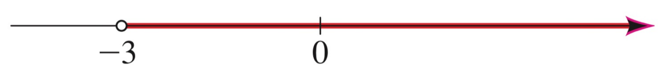
Express each interval in terms of inequalities, and then graph the interval.

(a) $[-1, 2) = \{x \mid -1 \le x < 2\}$

(b) $[1.5, 4] = \{x \mid 1.5 \le x \le 4\}$



(c) $(-3, \infty) = \{x \mid -3 < x\}$



EXAMPLE 4 **Graphing Inequalities**

(a) On the real number line, graph all numbers x for which $x > 4$.

(b) On the real number line, graph all numbers x for which $x \leq 5$.

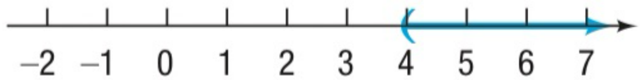


Figure 8 $x > 4$

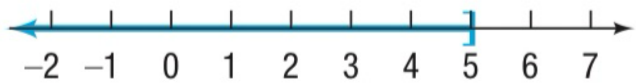


Figure 9 $x \leq 5$

In Problems 41–44, graph the numbers x on the real number line.

41. $x \geq -2$ **42.** $x < 4$ **43.** $x > -1$ **44.** $x \leq 7$

41. $[-2, \infty)$

42. $(-\infty, 4)$

43. $(-1, \infty)$

44. $(-\infty, 7]$

المتباينة الخطية :-

- $2x-5<7 \Rightarrow 2x<12$

  $x<6 \Rightarrow (-\infty, 6)$

- $-3x+4 \geq 1 \Rightarrow -3x \geq -3$

  $\Rightarrow x \leq 1 \Rightarrow (-\infty, 1]$

- $\frac{x}{2}+1 \geq -5 \Rightarrow \frac{x}{2} \geq -6$

  $x \geq -12 \qquad [-12, \infty)$

حل معادلات و متباينات القيمة المطلقة

إذا كان يساوي

$$|x| = c \Rightarrow \begin{cases} x = c \\ x = -c \end{cases}$$

شرط إن $c \geqslant 0$

مثال :-

$$|3 - x| = 5$$

$$3 - x = 5 \Rightarrow x = -2$$

$$3 - x = -5 \Rightarrow x = 8$$

الحل : $x = -2, 8$

إذا كانت متباينة (> أو <)

١) إذا كانت أصغر من (>)

$|x| < c \Rightarrow -c < x < c$

$|x-2| < 3$

$-3 < x-2 < 3$

$-1 < x < 5$

٢- إذا كانت أكبر من (<)

$|x| > c \Rightarrow \begin{cases} x > c \\ x < -c \end{cases}$

$|x+1| > 4$

$\Rightarrow x+1 > 4$ أو $x+1 < -4$

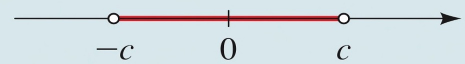
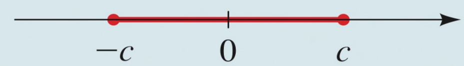
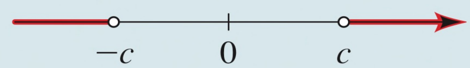
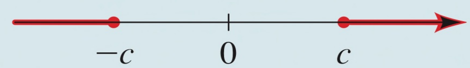
$x > +3$ أو $x < -5$

$(-\infty, -5) \cup (3, \infty)$

ملخص :-

$$|X| = C \quad \text{is equivalent to} \quad X = C \quad \text{or} \quad X = -C$$

PROPERTIES OF ABSOLUTE VALUE INEQUALITIES

Inequality	Equivalent form	Graph
1. $\lvert x \rvert < c$	$-c < x < c$	
2. $\lvert x \rvert \le c$	$-c \le x \le c$	
3. $\lvert x \rvert > c$	$x < -c$ or $c < x$	
4. $\lvert x \rvert \ge c$	$x \le -c$ or $c \le x$	

ملاحظة :-

$|x+3| = -2$ ✗

$|x+2| < -3$ ✗

تعويض داخل الدالة

التعويض يعني تاخذ قيمة معطاة للمتغير x وتضعها داخل الدالة $f(x)$ عشان تطلع الناتج.

الخطوات :-

١- اكتب الدالة كما هي.

٢- بدل كل ظهور لـ x بالقيمة المعطاة.

٣- احسب الناتج.

مثال :-

$$f(x) = 2x^2 + 3x - 1$$

$$f(2) = 2(2)^2 + 3(2) - 1 = 13$$

$$f(x) = \frac{e^{2x+1}}{x-3}$$

$$f(4) = \frac{e^{2(4)+1}}{4-3} = \frac{e^9}{1} = e^9$$

ما هو الـDomain ؟

الـDomain هو كل القيم المسموح بها لـ x بحيث تكون المعادلة مُعرفة و ما فيها قسمة على صفر أو جذر سالب أو داخل الـ Log عدد سالب.

---

١ ـ المقام $(\text{Denominator} \neq 0)$ ، لا يمكن للمقام أن يكون صفر.

مثال :- $f(x) = \frac{1}{x-2} \Rightarrow x-2=0 \Rightarrow x=2$ ✓

Domain: All real numbers excepet 2 $(-\infty,2)\cup(2,\infty)$ or $R/\{2\}$

---

٢ ـ الجذر الزوجي $(\text{Even Root} \geq 0)$

الجذر الزوجي مثل $\sqrt{x}$ يجب أن يكون ماتحت الجذر أكبر أو يساوي صفر.

مثال : $f(x)=\sqrt{x-5} \Rightarrow x-5 \geq 0 \Rightarrow x \geq 5$ ✓

Domain $[5,\infty)$

٣ـ الجذر الزوجي + مقام (Root in Denominator)

* ماتحت الجذر > ٥ فقط (ما يصير يساوي صفر)

مثال : $f(x) = \frac{1}{\sqrt{x-1}} \Rightarrow x-1>0 \Rightarrow x>1$ ✓

Domain $(1,\infty)$

٤ـ اللوغاريتم $(x)>0$, $(\log(x))$.

* لازم مابداخل اللوق > ٥

مثال : $f(x) = \ln(x+3) \Rightarrow x+3>0 \Rightarrow x>-3$

Domain $(-3,\infty)$

$\log(-5x+7) \Rightarrow -5x+7>0 \Rightarrow -5x>-7 \Rightarrow$

$\frac{-5x}{-5} < \frac{\cancel{-}7}{\cancel{-}5} \Rightarrow x < \frac{7}{5}$ Domain $(-\infty, \frac{7}{5})$

٥ـ القيمة المطلقة (No Restriction).

الدالة $|x|$ معرفة على جميع الأعداد الحقيقية.

مثال :

$f(x) = |x-1| \Rightarrow (-\infty,\infty)$

* دالة الأسية :- * No restriction *

e^x , 2^x , π^x , -10^{x^2} -----

Domain: $(-\infty, \infty)$

* $\sin x$, $\cos x$ * No restriction *

Domain: $(-\infty, \infty)$

* $\sin^{-1} x$, $\cos^{-1} x$ *

Domain: $[-1, 1]$

٦ـ الدوال المركبة (Combine All Rules)

مثال: $f(x) = \dfrac{\sqrt{x-2}}{x^2-9}$

١ـ من الجذر: $x-2 \geq 0 \Rightarrow x \geq 2$

٢ـ من المقام: $x^2-9 \neq 0 \Rightarrow x \neq 3, -3$

Combine: $x \geq 2$ and $x \neq 3 \Rightarrow [2,3) \cup (3,\infty)$

الحالة	القاعدة	مثال
مقام	المقام $\neq 0$	$\frac{1}{x}$
جذر زوجي	ماداخل الجذر ≥ 0	$\sqrt{x-1}$
جذر و مقام	ماداخل الجذر > 0	$\frac{1}{\sqrt{x}}$
Log	ماداخل اللوق > 0	$\ln(x)$
مطلق	معرف دائماً	$\lvert x \rvert$
دالة أسية	معرفة دائماً	$5^x , e^x$
دالة مثلثية	معرفة دائماً	$\sin x , \cos x$
دالة مثلثية عكسية	$[-1, 1]$	$\sin^{-1}x , \cos^{-1}x$

ثمرة

Thamrah

Kuwait University

Calculus 1 – Limits

(Section 1.2 & 1.4 & 1.5)

For Contact and Support:

YouTube: Precalculusq8 Twitter: Precalculusq8

Outlines :-

1- Types of function.

2- Exponential & log properties.

3- Domain of functions.

4- One to One functions.

5- Range & Inverse of functions.

6- Trigonometric identities & Simplify.

7- Simplify the Trigonometric.

1) Type of functions

- $f(x) = x^2 + 3x + 2$

- $f(x) = \dfrac{x}{x+1}$

- $f(x) = |x|$

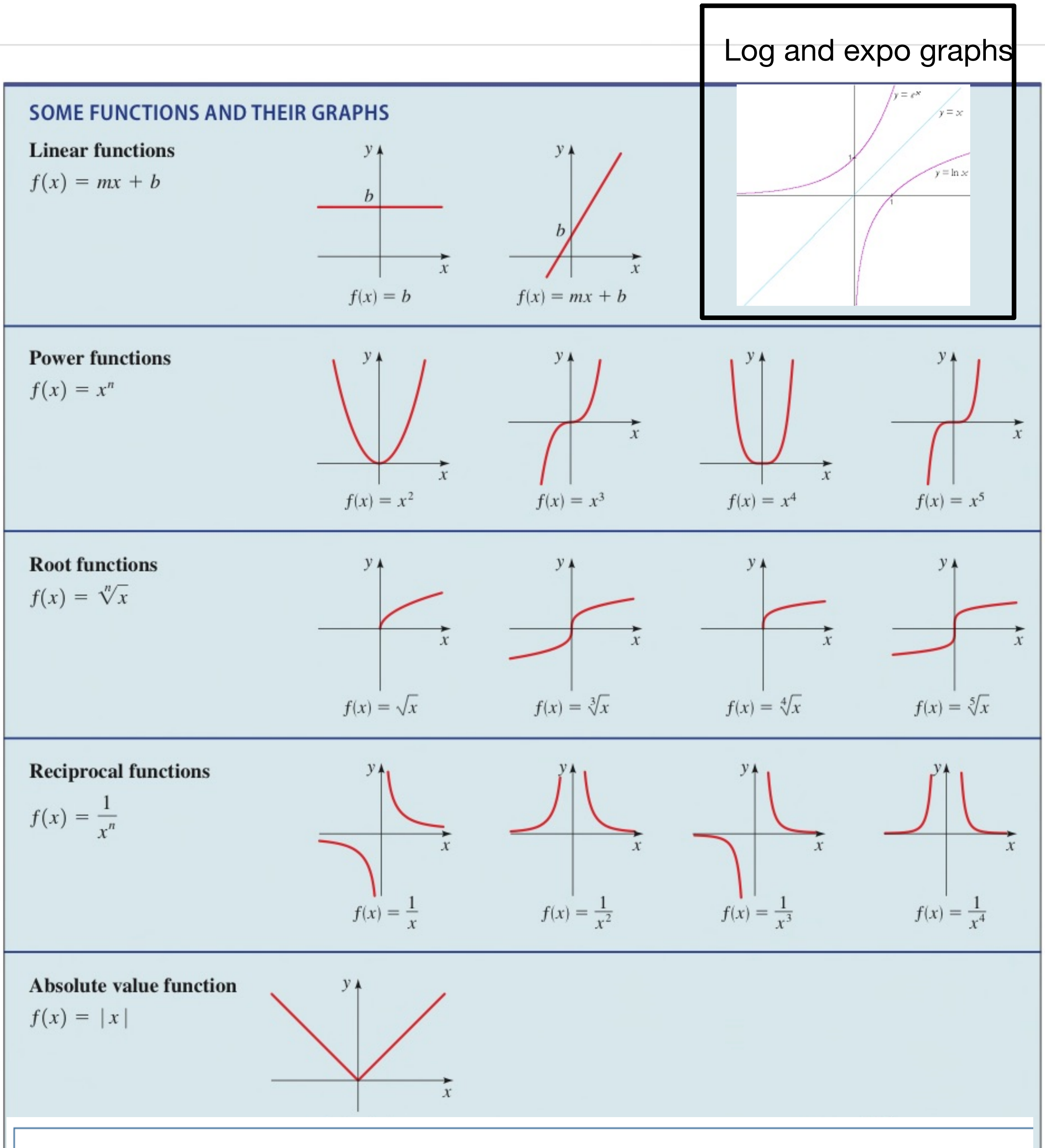
- $f(x) = \sqrt{x}$

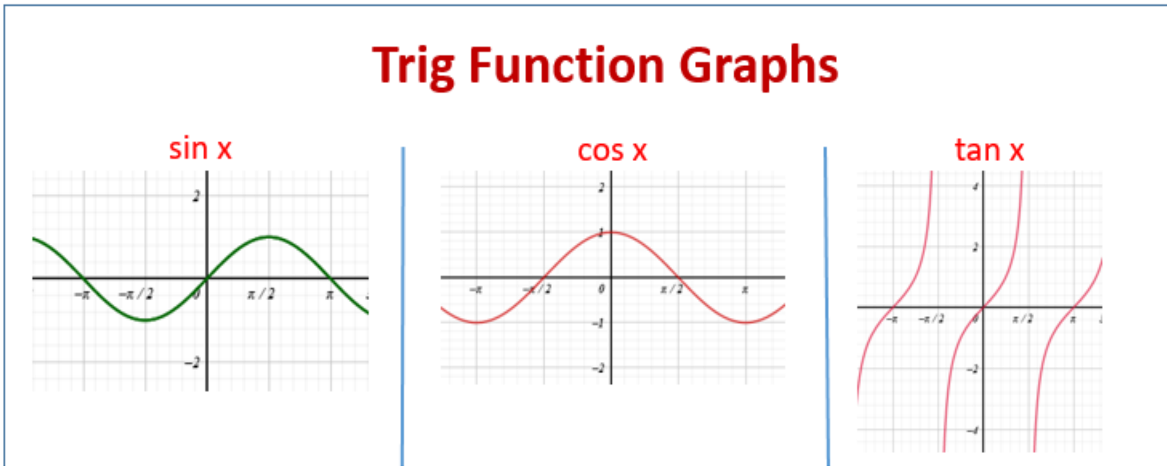
- $f(x) = e^x$

- $f(x) = \log$

- $f(x) = \text{Sin}, \text{Cos}, \tan$

رسمات الدوال





1–2 Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

1. (a) $f(x) = \log_2 x$ (b) $g(x) = \sqrt[4]{x}$

(c) $h(x) = \dfrac{2x^3}{1 - x^2}$ (d) $u(t) = 1 - 1.1t + 2.54t^2$

(e) $v(t) = 5^t$ (f) $w(\theta) = \sin\theta \cos^2\theta$

2. (a) $y = \pi^x$ (b) $y = x^\pi$

(c) $y = x^2(2 - x^3)$ (d) $y = \tan t - \cos t$

(e) $y = \dfrac{s}{1 + s}$ (f) $y = \dfrac{\sqrt{x^3 - 1}}{1 + \sqrt[3]{x}}$

2 - evaluating function.

1.4 EXERCISES

1–4 Use the Law of Exponents to rewrite and simplify the expression.

1. (a) $\dfrac{4^{-3}}{2^{-8}}$ (b) $\dfrac{1}{\sqrt[3]{x^4}}$

2. (a) $8^{4/3}$ (b) $x(3x^2)^3$

3. (a) $b^8(2b)^4$

4. (a) $\dfrac{x^{2n} \cdot x^{3n-1}}{x^{n+2}}$ (b) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$

23. If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x\left(\frac{5^h - 1}{h}\right)$$

Laws of Logarithms

EXAMPLE 1 ■ **Using the Laws of Logarithms to Evaluate Expressions**

Evaluate each expression.

(a) $\log_4 2 + \log_4 32$

(b) $\log_2 80 - \log_2 5$

(c) $-\frac{1}{3}\log 8$

Law

1. $\log_a(AB) = \log_a A + \log_a B$

2. $\log_a\left(\dfrac{A}{B}\right) = \log_a A - \log_a B$

3. $\log_a(A^C) = C\log_a A$

$$\frac{\log 6}{\log 2} \cancel{=} \log\left(\frac{6}{2}\right) \qquad \text{and} \qquad (\log_2 x)^3 \cancel{=} 3\log_2 x$$

$$\log_a(x + y) \cancel{=} \log_a x + \log_a y$$

39–41 Express the given quantity as a single logarithm.

39. $\ln 10 + 2 \ln 5$

40. $\ln b + 2 \ln c - 3 \ln d$

41. $\frac{1}{3} \ln(x + 2)^3 + \frac{1}{2} [\ln x - \ln(x^2 + 3x + 2)^2]$

39)

40)

39–41 Express the given quantity as a single logarithm.

✓ **39.** $\ln 10 + 2\ln 5$

✓ **40.** $\ln b + 2\ln c - 3\ln d$

41. $\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}[\ln x - \ln(x^2+3x+2)^2]$

35–38 Find the exact value of each expression.

35. (a) $\log_2 32$ (b) $\log_8 2$

EXAMPLE 8 Solve the equation $e^{5-3x} = 10$.

SOLUTION

The following formula shows that logarithms with any base can be expressed in terms of the natural logarithm.

10 **Change of Base Formula** For any positive number b ($b \neq 1$), we have

$$\log_b x = \frac{\ln x}{\ln b}$$

51–54 Solve each equation for x.

51. (a) $e^{7-4x} = 6$ (b) $\ln(3x - 10) = 2$

52. (a) $\ln(x^2 - 1) = 3$ (b) $e^{2x} - 3e^x + 2 = 0$

53. (a) $2^{x-5} = 3$ (b) $\ln x + \ln(x - 1) = 1$

51)

51–54 Solve each equation for x.

51. (a) $e^{7-4x} = 6$ (b) $\ln(3x - 10) = 2$

52. (a) $\ln(x^2 - 1) = 3$ (b) $e^{2x} - 3e^x + 2 = 0$

53. (a) $2^{x-5} = 3$ (b) $\ln x + \ln(x - 1) = 1$

52)

51–54 Solve each equation for x.

51. (a) $e^{7-4x} = 6$ (b) $\ln(3x - 10) = 2$

52. (a) $\ln(x^2 - 1) = 3$ (b) $e^{2x} - 3e^x + 2 = 0$

53. (a) $2^{x-5} = 3$ (b) $\ln x + \ln(x - 1) = 1$

53)

3) Domain

الحالة	القاعدة	مثال
مقام	المقام $\neq 0$	$\frac{1}{x}$
جذر زوجي	ماداخل الجذر ≥ 0	$\sqrt{x-1}$
جذر و مقام	ماداخل الجذر > 0	$\frac{1}{\sqrt{x}}$
Log	ماداخل اللوق > 0	$\ln(x)$
مطلق	معرف دائماً	$\lvert x \rvert$
دالة أسية	معرفة دائماً	$5^x , e^x$
دالة مثلثية	معرفة دائماً	$\sin^x , \cos^x$
دالة مثلثية عكسية	$[-1, 1]$	$\sin^{-1}x , \cos^{-1}x$

19–20 Find the domain of each function.

19. (a) $f(x) = \dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$ (b) $f(x) = \dfrac{1 + x}{e^{\cos x}}$

20. (a) $g(t) = \sqrt{10^t - 100}$ (b) $g(t) = \sin(e^t - 1)$

19–20 Find the domain of each function.

✓ **19.** (a) $f(x) = \dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$ ✓ (b) $f(x) = \dfrac{1 + x}{e^{\cos x}}$

20. (a) $g(t) = \sqrt{10^t - 100}$ (b) $g(t) = \sin(e^t - 1)$

4) One to One Function

* كل قيمة x مختلفة تعطي قيمة y مختلفة

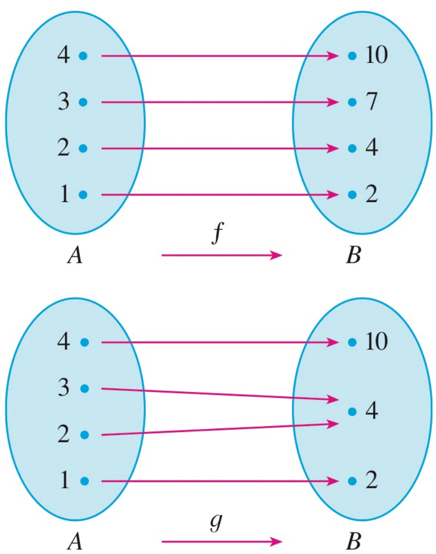
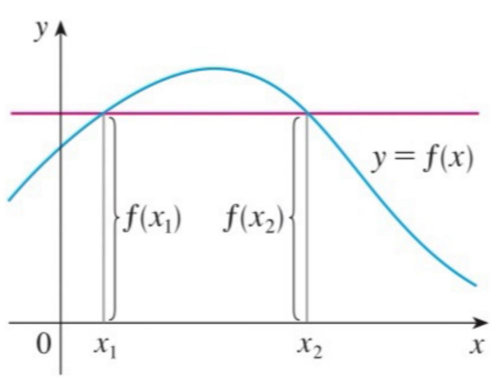


FIGURE 1
f is one-to-one; g is not.

1 Definition A function f is called a **one-to-one function** if it never takes on the same value twice; that is,

$$f(x_1) \neq f(x_2) \qquad \text{whenever } x_1 \neq x_2$$



* إذا قطع الخط الأفقي أكثر من مرة ⇐ الدالة
not one_to_one

Horizontal Line Test A function is one-to-one if and only if no horizontal line intersects its graph more than once.

طريقة الحل من خلال المعادلة

١) افرض $f(x_1) = f(x_2)$

* إذا الحل عطاني $x_1 = x_2$

∴ الدالة One-to-One

* إذا الحل عطاني $x_1 = \pm x_2$

∴ الدالة not One-to-One

طريقة الحل من خلال الرسم :-

١) نستخدم اختبار الأفقي

إذا أي خط أفقي يقطع الرسم في نقطة واحدة

فقط ⇐ الدالة One-to-One

إذا يقطع في أكثر من نقطة ⇐

not One-to-One .

3–14 Determine whether it is one-to-one.

9. $f(x) = 2x - 3$

10. $f(x) = x^4 - 16$

EXAMPLE 1 Is the function $f(x) = x^3$ one-to-one?

EXAMPLE 2 Is the function $g(x) = x^2$ one-to-one?

2 Definition Let f be a one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \iff f(x) = y$$

for any y in B.

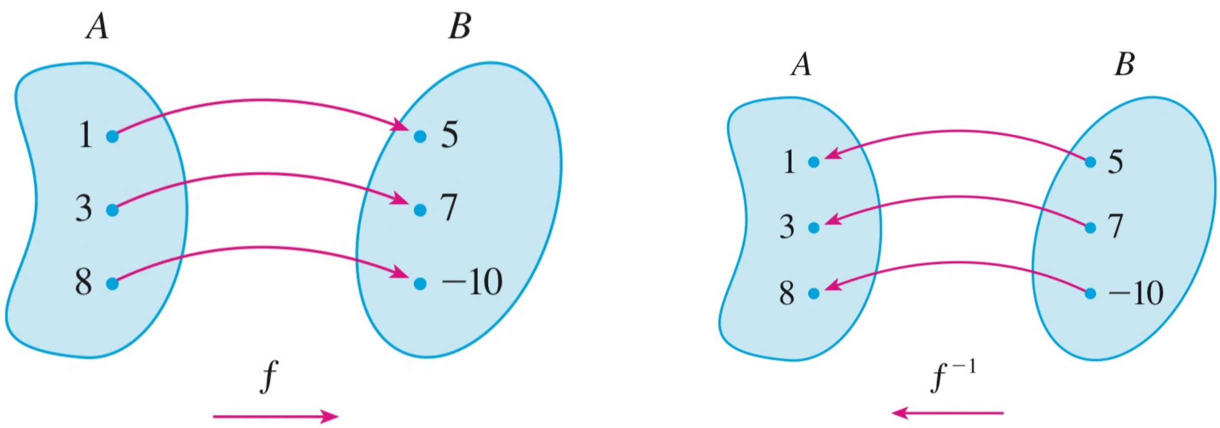


FIGURE 6
The inverse function reverses inputs and outputs.

$$\text{domain of } f^{-1} = \text{range of } f$$

$$\text{range of } f^{-1} = \text{domain of } f$$

For example, the inverse function of $f(x) = x^3$ is $f^{-1}(x) = x^{1/3}$ because if $y = x^3$, then

$$f^{-1}(y) = f^{-1}(x^3) = (x^3)^{1/3} = x$$

CAUTION Do not mistake the -1 in f^{-1} for an exponent. Thus

$$f^{-1}(x) \quad \text{does } \textit{not} \text{ mean} \quad \frac{1}{f(x)}$$

The reciprocal $1/f(x)$ could, however, be written as $[f(x)]^{-1}$.

3–14 A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

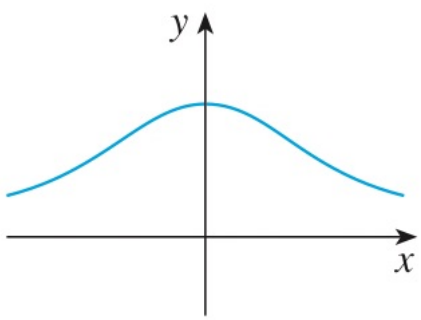
3.

x	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

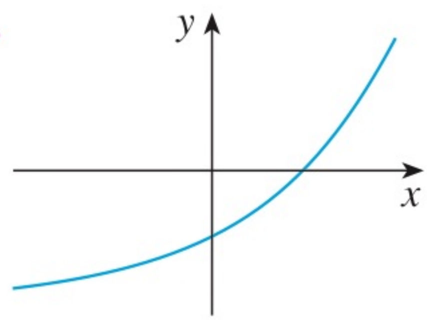
4.

x	1	2	3	4	5	6
$f(x)$	1.0	1.9	2.8	3.5	3.1	2.9

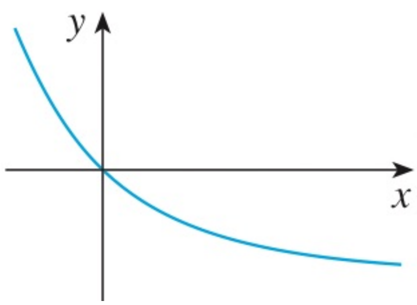
5.



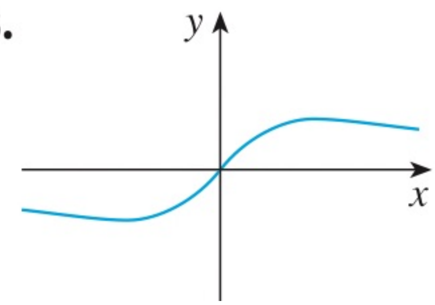
6.



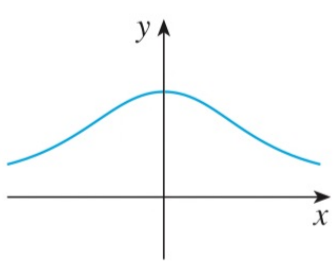
7.



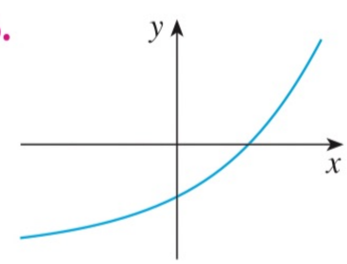
8.



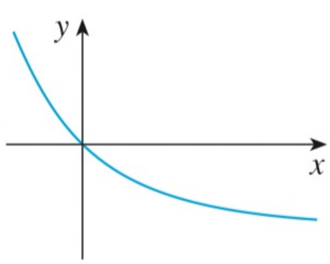
5.



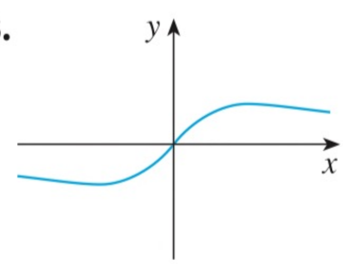
6.



7.



8.



EXAMPLE 3 If $f(1) = 5$, $f(3) = 7$, and $f(8) = -10$, find $f^{-1}(7)$, $f^{-1}(5)$, and $f^{-1}(-10)$.

cancellation equations:

$$f^{-1}(x) = y \iff f(y) = x$$

$$f^{-1}(f(x)) = x \quad \text{for every } x \text{ in } A$$

$$f(f^{-1}(x)) = x \quad \text{for every } x \text{ in } B$$

5 How to Find the Inverse Function of a One-to-One Function f

STEP 1 Write $y = f(x)$.

STEP 2 Solve this equation for x in terms of y (if possible).

STEP 3 To express f^{-1} as a function of x, interchange x and y. The resulting equation is $y = f^{-1}(x)$.

EXAMPLE 4 Find the inverse function of $f(x) = x^3 + 2$.

2. [5 + 5 = 10 pts.] The graph of f is given below.

(a) Explain why f is one-to-one.

(b) Find $f^{-1}(2)$.

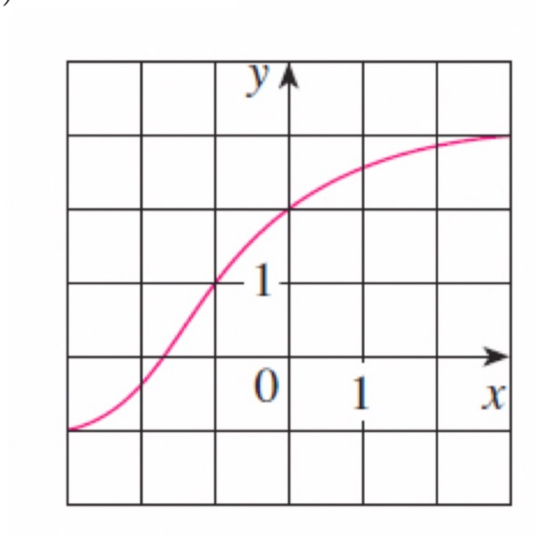


Figure 2: The graph of $y = f(x)$.

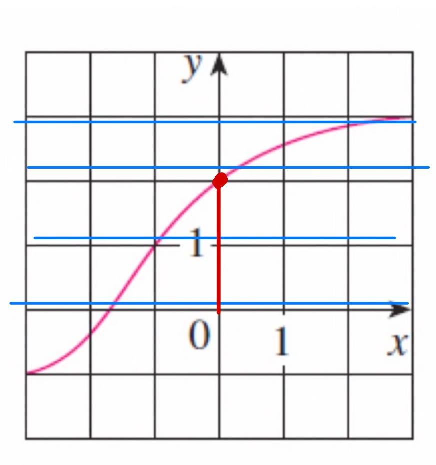


Figure 2: The graph of $y = f(x)$.

في مسائل الرسمة لازم تذكر سبب ليش one to one أو not one to one

21–26 Find a formula for the inverse of the function.

21. $f(x) = 1 + \sqrt{2 + 3x}$

22. $f(x) = \dfrac{4x - 1}{2x + 3}$

21–26 Find a formula for the inverse of the function.

21. $f(x) = 1 + \sqrt{2 + 3x}$

22. $f(x) = \dfrac{4x - 1}{2x + 3}$

21–26 Find a formula for the inverse of the function.

23. $f(x) = e^{2x-1}$

25. $y = \ln(x + 3)$

3. [10 pts.] Let $f(x) = \ln\left(\dfrac{x}{2x-1}\right)$. Find $f^{-1}(x)$.

21–26 Find a formula for the inverse of the function.

26. $y = \dfrac{1 - e^{-x}}{1 + e^{-x}}$

5. [10 pts.] Let $f(x) = e^{x+1} + 2$. Find $f^{-1}(x)$.

3. [10 pts.] Let $f(x) = \ln(2^x - 1)$. Find $f^{-1}(x)$.

6_range of the function :-

* هو جميع قيم x الناتجة من الدالة

لما نطبقها على الـ Domain .

* لإيجاد Range :-

١ـ أكتب $y = f(x)$

٢ـ حاول تعبر من الـ x بدلالة y "$f^{-1}(x)$"

٣ـ القيم المسموحة لـ y بعد الحل هي

الـ Range .

مثال :-

$f(x) = x^2$

$y = x^2$

$x = \sqrt{y}$

$f^{-1}(x) = \sqrt{x}$

Range of $f(x) = (0, \infty) =$ Domain of $f^{-1}(x)$.

2. [5 + 5 + 5 = 15 pts.] Let $f(x) = \dfrac{x+1}{2x+1}$.

(a) Show that f is one-to-one.

(b) Find the inverse function of f.

(c) Find the domain and range of f.

Let $f(x) = \dfrac{e^{x}}{1+2e^{x}}$, Find $f^{-1}(x)$

Domain of $f(x)$:-

57. (a) Find the domain of $f(x) = \ln(e^x - 3)$.
(b) Find f^{-1} and its domain.

3. [10 + 5 = 15 pts.] Let $f(x) = \ln\left(\frac{2}{x} + 3\right)$.

(a) Find the inverse function of f.

(b) Find the range of f.

4. [5+5 = 10 pts.] Let $f(x) = 3^x + 1$.

a) Find $f^{-1}(x)$.

b) Find the range of f.

4. $(10 + 5 = 15$ pts) Let $f(x) = \ln(2x + 3)$.

(a) Find $f^{-1}(x)$.

(b) Find the domain and the range of the function f.

Inverse Trigonometric Functions

When we try to find the inverse trigonometric functions, we have a slight difficulty: Because the trigonometric functions are not one-to-one, they don't have inverse functions. The difficulty is overcome by restricting the domains of these functions so that they become one-to-one.

You can see from Figure 17 that the sine function $y = \sin x$ is not one-to-one (use the Horizontal Line Test). But the function $f(x) = \sin x, -\pi/2 \leqslant x \leqslant \pi/2$, *is* one-to-one (see Figure 18). The inverse function of this restricted sine function f exists and is denoted by $\sin^{-1}$ or arcsin. It is called the **inverse sine function** or the **arcsine function**.

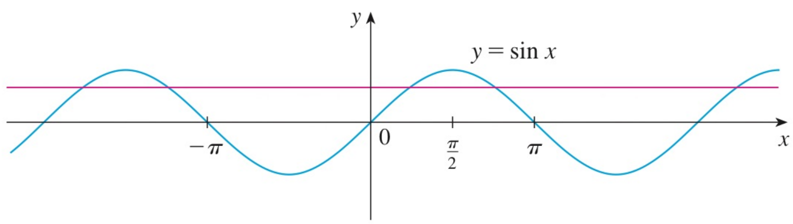


FIGURE 17

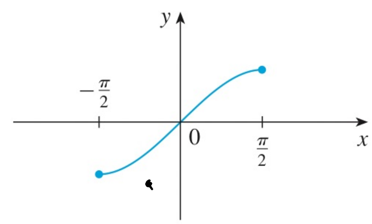


FIGURE 18
$y = \sin x, -\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$

THE INVERSE SINE, INVERSE COSINE, AND INVERSE TANGENT FUNCTIONS

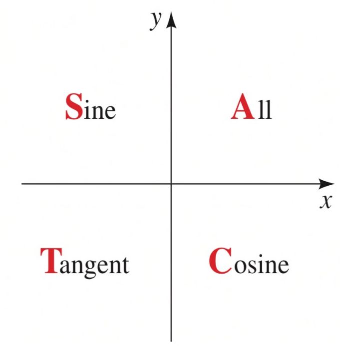
The sine, cosine, and tangent functions on the restricted domains $[-\pi/2, \pi/2]$, $[0, \pi]$, and $(-\pi/2, \pi/2)$, respectively, are one-to one and so have inverses. The inverse functions have domain and range as follows.

Function	Domain	Range
$\sin^{-1}$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}$	$\mathbb{R}$	$(-\pi/2, \pi/2)$

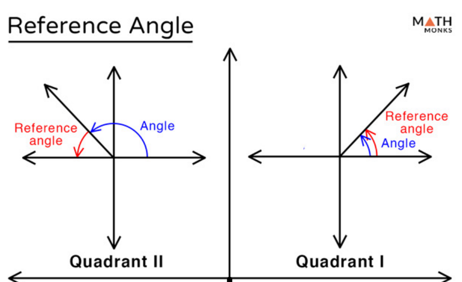
The functions $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ are sometimes called **arcsine**, **arccosine**, and **arctangent**, respectively.

inverse Trigo   معلومات مهمة

١) اعرف الاشارات في الأرباع



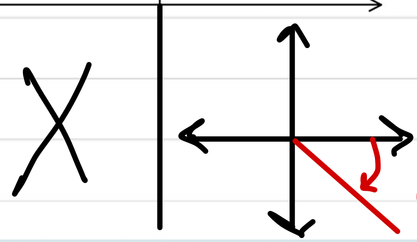
٢) اعرف الزاوية المرجعية ($\bar{\theta}$)



١) الربع الأول ⇐ $\theta = \bar{\theta}$

٢) الربع الثاني ⇐ $\theta = \pi - \bar{\theta}$

٣) الربع الثالث ⇐ $\theta = - \bar{\theta}$



Quad VI

٣) اعرف القيم المحفوظة

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	—

٤) Range of inverse

Function	Domain	Range
$\sin^{-1}$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}$	$\mathbb{R}$	$(-\pi/2, \pi/2)$

خطوات إيجاد قيم inverse Trig

١) أوجد Range الدالة المثلثية المعكوسة

$\sin^{-1} x \Rightarrow$ الرابع + الأول الربع $\Rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\cos^{-1} x \Rightarrow$ الثاني + الأول الربع $\Rightarrow [0, \pi]$

$\tan^{-1} x \Rightarrow$ الرابع + الأول الربع $\Rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

٢) حدد الزاوية المرجعية

$\bar{\theta} = 0°, 30°, 45°, 60°, 90°$ & in rad

٣) حدد الربع باستخدام إشارة القيمة (موجب / سالب)

٤) اكتب النتيجة θ in rad

١) الربع الأول $\bar{\theta} = \theta$

٢) الربع الثاني $\theta = \pi - \bar{\theta}$

٣) الربع الرابع $\theta = -\bar{\theta}$

EXAMPLE 1 ■ Evaluating Inverse Trigonometric Functions

Find the exact value.

(a) $\sin^{-1}\dfrac{\sqrt{3}}{2}$ **(b)** $\cos^{-1}\left(-\frac{1}{2}\right)$

63–68 Find the exact value of each expression.

63. (a) $\cos^{-1}(-1)$ (b) $\sin^{-1}(0.5)$

64. (a) $\tan^{-1}\sqrt{3}$ (b) $\arctan(-1)$

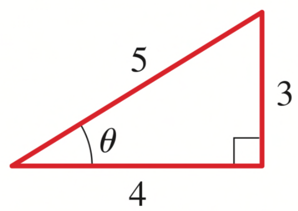
$*\tan^{-1}(1)$

65. (a) $\csc^{-1}\sqrt{2}$ (b) $\arcsin 1$

66. (a) $\sin^{-1}(-1/\sqrt{2})$ (b) $\cos^{-1}(\sqrt{3}/2)$

66. (a) $\sin^{-1}(-1/\sqrt{2})$ (b) $\cos^{-1}(\sqrt{3}/2)$

6 - Trigonometric identities & Simplify



$\sin\theta = \frac{3}{5}$

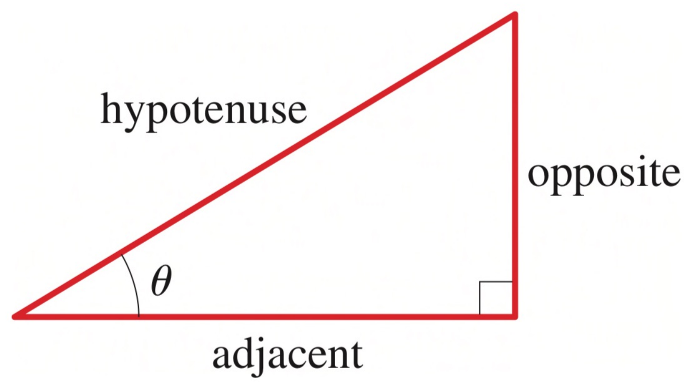


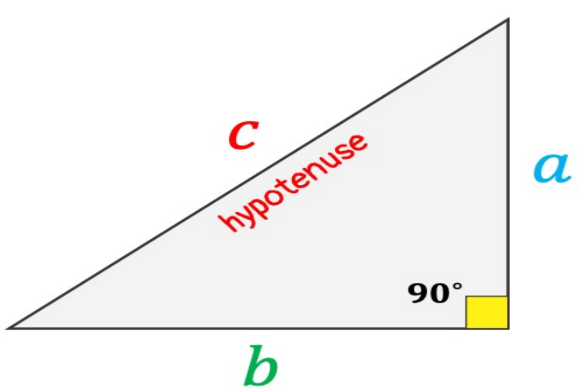
FIGURE 1

THE TRIGONOMETRIC RATIOS

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

PYTHAGOREAN THEOREM



$$\boxed{c^2 = a^2 + b^2}$$

- $c = \sqrt{a^2 + b^2}$
- $a = \sqrt{c^2 - b^2}$
- $b = \sqrt{c^2 - a^2}$

حل مسائل دوال المثلثية و الـ inverse لها :-

* prove: $\cos(\sin^{-1} x) = \sqrt{1-x^2}$

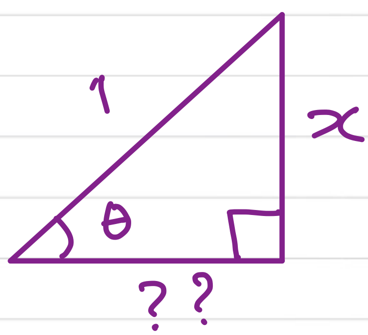
١) عيّن الزاوية

$\theta = \sin^{-1} x$

٢) حول النسبة المثلثية

$\sin\theta = \frac{x}{1}$

٣) ارسم مثلث قائم حسب النسبة



٤) أوجد الضلع المفقود بإستخدام فيثاغورث

$1^2 = ??^2 + x^2 \Rightarrow ?? = \sqrt{1-x^2}$

٥) اقرأ المطلوب :

$\cos\theta = \frac{??}{1} = \sqrt{1-x^2}$

69. Prove that $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$.

4. [10 pts.] Find the exact value of each expression

(a) $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$.

(b) $\log_{10} 4 + 2\log_{10} 5$.

1. [10 pts.] Prove that: $\sec(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$ for any x in $(-1, 1)$.

2. [10 pts.] Find the exact value of $\sin\left(\sec^{-1}\left(\frac{5}{3}\right)\right)$.

3. [5 pts.] Find the exact value of $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$.

7. $(5+5=10$ pts$)$

(a) Show that $\sin\left(\tan^{-1} x\right) = \dfrac{x}{\sqrt{1+x^2}}$.

(b) Find the exact value of $\sin\left(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$.

Find the exact value of

(a) $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$.

7_Simplify the Trigonometric.

70–72 Simplify the expression.

70. $\tan(\sin^{-1} x)$ **71.** $\sin(\tan^{-1} x)$