CLO	1,2	1,2	1,2	1	1,3	2	2	
Question	1 a	1 b	1 c	2	3	4	5	Total
Grade	$\overline{}$ 3	$\overline{}$ 3	$\overline{}$ 4	$\overline{}$ 10	$\overline{}$ 10	$\overline{}$ 10	$\overline{}$ 10	$\overline{}$ 50

UAEU College of Science كلية العلوم
Department of Mathematical Sciences

جامعة الإمارات العربية المتحدة
United Arab Emirates University
قسم علوم الرياضيات

Math 105
Calculus 1

Test 1-Form B - Spring 2018 Solution
Feb. 14, 2018

Student Name:................................ **Instructor: Dr. Muhammad Imran**

ID number:...................................... **Section number: 61**

Please write your answers in detail and show your work to get full mark. Giving short answers might cause you losing points.

1. Evaluate the following the limit and conclude if it has vertical or horizontal asymptotes.

(a)

$$\lim_{x\to\infty} \frac{5-3x+4x^2}{x^3+3x^2-x+2}$$

Sol:-

$$= \lim_{x\to\infty} \frac{\cancel{x^2}\left(5/x^2 - 3x/x^2 + 4\right)}{\cancel{x^3}\left(1 + 3x^2/x^3 - x/x^3 + 2/x^3\right)} = \lim_{x\to\infty} \frac{1}{x} \frac{\left(5/x^2 - \frac{3}{x} + 4\right)}{\left(1 + 3/x - 1/x^2 + 3/x^3\right)}$$

$$= \frac{\left(5/\infty - \frac{3}{\infty} + 4\right)}{\infty\left(1 + 3/\infty - 1/\infty + 3/\infty\right)} = 5/\infty = 0 \text{ Ans} \Rightarrow y = 0 \text{ is horizontal asymptote.}$$

(b) $\lim_{t\to 0} \frac{\sqrt{2-t}-\sqrt{2}}{t}$

Sol:-

$$= \lim_{t\to 0}\left(\frac{\sqrt{2-t}-\sqrt{2}}{t} \times \frac{\sqrt{2-t}+\sqrt{2}}{\sqrt{2-t}+\sqrt{2}}\right) = \lim_{t\to 0} \frac{\cancel{2}-t-\cancel{2}}{t\left(\sqrt{2-t}+\sqrt{2}\right)}$$

$$= \lim_{t\to 0} -\frac{\cancel{t}}{\cancel{t}\left(\sqrt{2-t}+\sqrt{2}\right)} = -\frac{1}{\sqrt{2}+\sqrt{2}} = -\frac{1}{2\sqrt{2}}$$

$\Rightarrow t = 0$ is Vertical Asymptotes.

(c) $\lim_{x\to\infty} \frac{\cos(1/x)}{x^6}$

Sol:-

Since

$$-1 \le \cos(1/x) \le 1$$

$$\Rightarrow -\frac{1}{x^6} \le x^6 \cos(1/x) \le \frac{1}{x^6}$$

$$\Rightarrow \lim_{x\to\infty} -\frac{1}{x^6} \le \lim_{x\to\infty} x^6 \cos(1/x) \le \lim_{x\to\infty} (1/x^6)$$

Since $\lim_{x\to\infty}\left(-\frac{1}{x^6}\right) = \lim_{x\to\infty} \frac{1}{x^6} = 0$

$\Longrightarrow$ by Squeeze theorem, $\lim_{x\to\infty} x^6 \cos(1/x^6) = 0$

$\Rightarrow y = 0$ is horizontal asymptotes.

& $x = 0$ is vertical asymptote.

2. For what value of a and b is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} x^3 - 6a & \text{if} \quad x < -3 \\ 3 & \text{if} \quad x = -3 \\ bx - a & \text{if} \quad x > -3 \end{cases}$$

Sol:- Since $f(x)$ is Continuous on $(-\infty, \infty)$

$\Rightarrow$ $f(x)$ is Continuous at $x = -3$

$$\Rightarrow \lim_{x \to 3} f(x) = f(x)$$

$$\Rightarrow \lim_{x \to -3^+} f(x) = \lim_{x \to -3^-} f(x) = f(-3)$$

Now

$$\lim_{x \to -3^+} f(x) = f(-3) \Rightarrow \lim_{x \to -3^+} (bx - a) = 3$$

$$\Rightarrow -3b - a = 3$$

$$\Rightarrow \boxed{a + 3b = -3} \longrightarrow (1)$$

Also

$$\lim_{x \to -3^-} f(x) = f(-3)$$

$$\Rightarrow \lim_{x \to -3^-} (x^3 - 6a) = 3$$

$$\Rightarrow (-3)^3 - 6a = 3$$

$$-27 - 6a = 3$$

$$-6a = 3 + 27$$

$$-6a = 30 \Rightarrow \boxed{a = -5}$$

$(1) \Rightarrow$

$$-5 + 3b = -3$$

$$3b = 5 + 3 \Rightarrow 3b = 8$$

$$\boxed{b = 8/3}$$

3. Use the Intermediate Value Theorem to show that there is a root of the given equation in the interval $[-2, 0]$.

$$x^5 - 2x^3 + x^2 + 2 = 0,$$

$$f(x) = x^5 - 2x^3 + x^2 + 2$$

$f(x)$ is continuous as it is polynomial of degree 5.

Also $f(-2) = (-2)^5 - 2(-2)^3 + (-2)^2 + 2$

$= -32 + 16 + 4 + 2 = -10 < 0$

$f(0) = 0 - 0 + 0 + 2 > 0$

$\Rightarrow$ By Intermediate Value theorem, $\exists$ a root of equ in interval $[-2, 0]$.

4. Differentiate the following function.

(a) $g(x) = (\frac{x^2+5}{x^2-4})^2$

by Chain Rule & Quotient Rule

Sol:- $g'(x) = 2\left(\frac{x^2+5}{x^2-4}\right)^{2-1} \frac{d}{dx}\left[\frac{x^2+5}{x^2-4}\right]$

$= 2\left(\frac{x^2+5}{x^2-4}\right)\left[\frac{(x^2-4)(2x)-(x^2+5)(2x}{(x^2-4)^2}\right]$

$= \frac{2\cdot 2x\,(x^2+5)}{x^2-4}\left[\frac{\cancel{x^2}-4-\cancel{x^2}-5}{(x^2-4)^2}\right] = \frac{-36(x^2+5)}{(x^2-4)^3}$ Ans

(b) $h(x) = 7^{\cos(3\pi x^2)}$

$\left[\frac{d}{dx}(a^x) = a^x \ln a\right]$

Sol:- $h'(x) = 7^{\cos(3\pi x^2)} \ln 7 \frac{d}{dx}\left[\cos(3\pi x^2)\right]$

$= 7^{\cos(3\pi x^2)} \ln 7 \left(-\sin(3\pi x^2)\right) \frac{d}{dx}(3\pi x^2)$

$= -7^{\cos(3\pi x^2)} \ln 7 \cdot \sin(3\pi x^2) \cdot 6x\pi \cdot$

Ans

5. Write the composite function in the form $f(g(x))$ and evaluate $u'(x)$

$$\cancel{u(x) = \frac{\cos(x)^2}{3\tan(x) - 1}} \qquad u(x) = \frac{\cos x}{3\cos x - 1}$$

Sol:-

$$f(x) = \frac{x}{3x-1}$$

$$g(x) = \cos x$$

$$u'(x) = \frac{(3\cos x - 1)\frac{d}{dx}(\cos x) - \cos x \frac{d}{dx}(3\cos x - 1)}{(3\cos x - 1)^2}$$ (Quotient Rule)

$$= \frac{-(3\cos x - 1)\sin x + \cos x(3\sin x)}{(3\cos x - 1)^2}$$

$$= \frac{\sin x\left[-3\cos x + 1 + \cos x\right]}{(3\cos x - 1)^2}$$

$$= \frac{\sin x\left[-2\cos x + 1\right]}{(3\cos x - 1)^2}$$ Ans