United Arab Emirates University

Faculty of Science

Department of Mathematical Sciences

MATH 105 CALCULUS I MIDTERM EXAM FALL 2024

Date and time : 8/10/2024, 7:00-8:00pm

Student Name: .....

Student ID: .....

Section: .....

Attendance Number: .....

Section	Instructor	Section	Instructor	Section	Instructor
01	A. Hamdan	51	A. Al Rawashdeh	56	M. Hajji
02	M. Syam	52	M. Hajji	57	A. Al Rawashdeh
03	A. Zaytouni	53	N. Chbili	58	N. Chbili
04	M. Syam	54	Y. El Khatib	59	A. Diene
05	F. Alawadhi	55	D. Diene	60	S. Ben Said

Questions	1	2 & 3	4 & 5	Total Grade
Mark	..... / 6	..... / 12	..... / 7	..... / **25**
CLO	2	2	1	

- Show all steps of your solution for each question (except for multiple-choice questions).
- Use only blue or black pen; do not use pencil or colored pens.
- Graphing and programmable calculators are not allowed.

Question 1. [**6 marks**] Evaluate the following limits. Show all details of your work.

(a) $\lim_{x\to 2} \frac{x-2}{x^2-5x+6}$. $= \frac{0}{0}$

$$\lim_{x\to 2} \frac{x-2}{(x-3)(x-2)} = \frac{1}{2-3} = -\frac{1}{1} = -1$$

(b) $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x^2-9}$. $= \frac{0}{0}$

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x^2-9} \times \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \frac{x+1-4}{(x-3)(x+3)(\sqrt{x+1}+2)}$$

$$\Rightarrow \frac{x-3}{(x-3)(x+3)(\sqrt{x+1}+2)} = \frac{1}{(3+3)(\sqrt{3+1}+2} = \frac{1}{(6)(4)}$$

(c) $\lim_{x\to\infty} \frac{\cos x}{x^2+1}$. (Use the Squeeze Theorem) $= \frac{1}{24}$

$$-1 \leq \cos x \leq 1$$

$$\frac{-1}{x^2+1} \leq \frac{\cos x}{x^2+1} \leq \frac{1}{x^2+1} \Rightarrow \lim_{x\to\infty} \frac{-1}{x^2+1} = \frac{-1}{\infty} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^2+1} = \frac{1}{\infty} = 0 \quad \therefore \text{By S.T}$$

$$\lim_{x\to\infty} \frac{\cos x}{x^2+1} = 0$$

Question 2. [6 marks] Find the derivative $\frac{dy}{dx}$ in each of the following cases (Do not simplify).

(a) $y = \frac{x^3}{x^2+1}$.

$$y' = \frac{(3x^2)(x^2+1) - [2x * x^3]}{(x^2+1)^2}$$

(b) $y = x^2 e^{3x}$

$$y' = 2xe^{3x} + 3e^{3x}x^2$$

(c) $y = \sin^{-1}(2x+1)$.

$$y' = \frac{2}{\sqrt{1-(2x+1)^2}}$$

Question 3. [6 marks]

(a) Use Logarithmic Differentiation to differentiate $y = x^{\cos x}$.

$$\ln y = \cos x \ln x$$

$$\frac{y'}{y} = -\sin x \ln x + \frac{\cos x}{x}$$

$$y' = y\left(-\sin x \ln x + \frac{\cos x}{x}\right)$$

$$y' = x^{\cos x}\left(-\sin x \ln x + \frac{\cos x}{x}\right)$$

(b) Write an equation of the tangent line to the curve $y^2 + 2x - 3x^2y = 0$ at the point $P(1,2)$.

$$2yy' + 2 - 3(2xy + 3x^2y') = 0$$

$$2yy' + 2 - 6xy - 3x^2y' = 0$$

$$2yy' - 3x^2y' = 6xy - 2$$

$$y'(2y - 3x^2) = 6xy - 2$$

$$y' = \frac{6xy - 2}{2y - 3x^2} \quad @\ P(1,2)$$

$$\Rightarrow y' = \frac{12 - 2}{4 - 3} = \frac{10}{1} = 10$$

$$y - 2 = 10(x - 1)$$

$$y = 10x - 8$$

Question 4. [2 marks] Use the Intermediate Value Theorem to show that the equation $\ln(x) + x - 2 = 0$ has a solution in the interval $(1, 3)$.

let $f(x) = \ln(x) + x - 2 = 0$

. f is continuous $(1,3)$ [log, poly]

. $f(1) = \ln(1) + 1 - 2 = -1 < 0$

. $f(3) = \ln 3 + 3 - 2 = \ln 3 + 1 > 0$

since $f(1) < 0 < f(3)$ and f is cont, by IVT there exist $c \in (1,3)$ with $f(c) = 0$

Question 5. **[5 marks]** For each of the questions below, put a circle around the correct answer.

1. For which values of a is the function $f(x)$ continuous everywhere?

$$f(x) = \begin{cases} e^x + x^2 & \text{if } x \leq 0 \\ x + a - 3 & \text{if } x > 0 \end{cases}$$

(A) $a = -1$.

(B) $a = 1$ and $a = -1$.

✓ (C) $a = 4$.

(D) $a \neq 0$.

$1 + 0 = a - 3$

$a = 4$

2. Which of the following functions has the line $y = 2$ as a horizontal asymptote.

(A) $\frac{2x + x^2}{x^2 + 1}$ (B) $\frac{2x^2}{x^2 + 2}$ (C) $\frac{x^3}{x - 2}$ (D) $\frac{2x}{x^2 + x + 1}$

3. $\lim_{x \to 1^+} \frac{-x}{|x - 1|} = ?$ $\Rightarrow \frac{-1^+}{|1^+ - 1|} = \frac{-1^+}{0^+} = -\infty$

(A) 0 (B) -1 (C) ∞ (D) $-\infty$

4. The derivative of order 3 of $f(x) = 10^x$; $f^{(3)}(x) = ?$

(A) 10^{x-3} (B) $(\ln 10)10^x$ (C) $(\ln 10)^3 10^x$ (D) $x^3 10^{x-3}$

$y' = 10^x (\ln 10) \Rightarrow \ln 10^3$

5. Given that $f(x) + x(f(x))^2 - 4x = 1$, then $f'(0) = ?$

(A) 0 (B) 1 (C) 2 (D) 3

find $f(0) \Rightarrow f(0) + 0 f(0)^2 - 4(0) = 1$

$\Rightarrow f(0) + 0 - 0 = 1 \Rightarrow f(0) = 1$

diff equation from the question:

$f'(x) + [f(x)]^2 + 2(f(x)) f'(x)\, x - 4 = 0$

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$f'(x)[1 + 2f(x)\, x] + [f(x)]^2 - 4 = 0$

$$f'(x) = \frac{-[f(x)]^2 + 4}{1 + 2f(x)\, x}$$

$$f'(0) = \frac{-[1]^2 + 4}{1 + 2(1)(0)} = \frac{3}{1} = 3$$